

3-5 Equations with the Variable on Both Sides

Objective: To solve equations with the variable on both sides.

Example 1 Solve $5x = 2x + 15$.

Solution $5x - 2x = 2x + 15 - 2x$ Subtract $2x$ from each side. $3x = 15$
 $x = 5$
 The solution set is $\{5\}$.

Check: $5(5) \stackrel{?}{=} 2(5) + 15$
 $25 \stackrel{?}{=} 10 + 15$
 $25 = 25 \checkmark$

Example 2 Solve $4x = 30 - x$.

Solution $4x + x = 30 - x + x$ Add x to each side.
 $5x = 30$
 $x = 6$ The solution set is $\{6\}$.

Solve.

1. $5n = 3n + 8$ {4} 2. $7a = 2a + 30$ {6} 3. $y = 20 - 3y$ {5} 4. $3b = 80 - 5b$ {10}
5. $10n = 36 - 2n$ {3} 6. $2x = 20 - 8x$ {2} 7. $21a = 56 + 7a$ {4} 8. $30 + 6x = 12x$ {5}
9. $-9a = -12a - 45$ {15} 10. $33c + 60 = 21c$ {-5} 11. $72 - 4n = -22n$ {-4} 12. $-11a = -12a - 21$ {-21}

Example 3 Solve $\frac{2}{5}x + 12 = x$.

Solution $\frac{2}{5}x + 12 - \frac{2}{5}x = x - \frac{2}{5}x$ Subtract $\frac{2}{5}x$ from each side.
 $12 = \frac{5}{5}x - \frac{2}{5}x$ Rewrite $1x$ as $\frac{5}{5}x$.
 $12 = \frac{3}{5}x$
 $\frac{5}{3} \cdot \frac{12}{1} = \frac{5}{3} \left(\frac{3}{5}x \right)$ Multiply each side by $\frac{5}{3}$, the reciprocal of $\frac{3}{5}$.
 $20 = x$ The solution set is $\{20\}$.

Example 4 Solve $\frac{6+x}{3} = x$.

Solution $3\left(\frac{6+x}{3}\right) = 3 \cdot x$ Multiply each side by 3, the reciprocal of $\frac{1}{3}$.
 $6 + x = 3x$
 $6 + x - x = 3x - x$ Subtract x from each side.
 $6 = 2x$
 $3 = x$ The solution set is $\{3\}$.

3-5 Equations with the Variable on Both Sides (continued)

Solve.

13. $\frac{2}{3}x - 5 = x$ {-15} 14. $\frac{3}{4}x - 8 = x$ {-32} 15. $x = \frac{1}{2}x + 7$ {14} 16. $x = \frac{4}{5}x - 9$ {-45}
17. $\frac{x-2}{3} = x$ {-1} 18. $\frac{3+y}{4} = y$ {1} 19. $y = \frac{7-2y}{5}$ {1} 20. $x = \frac{9+x}{4}$ {3}

Vocabulary

Empty set or null set The set with no members.

Identity An equation that is true for every value of the variable(s).

Symbol ϕ (empty set, or the null set)

CAUTION An equation may have no solution, or it may be satisfied by every real number.

Example 5 Solve: a. $5(a-2) - 3 = 3a + 7 + 2a$ b. $\frac{1}{3}(24x - 15) = 8x - 5$

Solution a. $5a - 10 - 3 = 5a + 7$ b. $8x - 5 = 8x - 5$ — Identity
 $5a - 13 = 5a + 7$ An identity is true for every value of the variable.
 $-13 = 7$ — False
 The equation has *no solution*. The solution set is {real numbers}.

Solve each equation. If the equation is an identity or if it has no solution, write *identity* or *no solution*.

21. $2(x-3) = 5x$ {-2} 22. $4(y-5) = 9y$ {-4} 23. $3n = 6(3-n)$ {2}
24. $-3m = 5(2-m)$ {5} 25. $2(a-1) = 2a + 3$ No solution 26. $\frac{1}{4}(28x-8) = 7x-2$ Identity
27. $\frac{1}{3}(3x-3) + 2 = 2x$ {1} 28. $4(a-1) - 5 = 3a + 7$ {16} 29. $3(5+y) - y = 2y + 15$ Identity
30. $4a + 7 + a = 3(a-1)$ {-5} 31. $\frac{3n-15}{4} = 2n$ {-3} 32. $\frac{2n-9}{2} = n$ No solution

Mixed Review Exercises

Simplify.

1. $3 + \left(-\frac{1}{3}\right) + \left(-\frac{5}{3}\right)$ 1 2. $-2\frac{3}{4} + 1\frac{1}{4} - 1\frac{1}{2}$ 3. $-115 - (-10)$ -105
4. $15x + (-3x) - 2$ $12x - 2$ 5. $-4y + 5 + 18y + 23$ $14y + 28$ 6. $6(-2)(-5)(-4)$ -240

Solve.

7. $-2 - x = 5$ {-7} 8. $4 + (1+k) = 2$ {-3} 9. $3x = -276$ {-92}
10. $\frac{1}{2}x = 3\frac{1}{2}$ {7} 11. $\frac{x}{6} = 7$ {42} 12. $-10\frac{2}{3} = -\frac{1}{3}x$ {32}